FAST SIMULATION OF LARGE-SCALE PLANAR CIRCUITS USING AN OBJECT-ORIENTED SPARSE SOLVER

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ABSTRACT

This paper presents a novel approach based on sparse matrix techniques to the assembly and inversion of linear systems that result from numerical modeling of planar structures using the method of moments (MoM). In order to render moment matrices as sparse as possible, one can take advantage of different types and levels of sparseness, both natural and approximate. For efficient storage and inversion of sparse linear systems, a block decomposition of the moment matrix is proposed. The software implementation of this approach in a CAD package demands full utilization of object-oriented programming techniques.

I. INTRODUCTION

The method of moments offers one of the most accurate and efficient tools for the numerical modeling of planar microwave circuits. Integral-based formulations often result in linear systems of much smaller size and better numerical stability when compared to their differential-based counterparts. In spite of its inherent computational superiority in handling planar structures, the method of moments does not enjoy the natural sparseness of linear systems as in differential-based techniques. In the treatment of large-scale circuits, the processes of matrix fill and system inversion both become exhaustively time consuming, with the latter being the dominant computational process. In such cases, matrix sparseness is a crucial factor in view of both memory usage and computation time.

In recent years, extensive research efforts have been undertaken to develop accelerating tools for moment method solutions. Based on the theory of multiresolution analysis, wavelet expansions or transformations have been proposed to render moment matrices sparse after performing a thresholding process [1]-[2]. Fast multiple multipole (FMM) techniques [3] and impedance matrix localization [4] are among other methods proposed for speeding up the method of moments. All of these techniques are based on mathematical developments that address the formulation of the problem in a direct way.

In this paper, the problem of matrix sparseness is viewed in the light of efficient code programming. There are different types and levels of sparseness, both natural and approximate. These include sparseness due to decoupling of integral equations, sparseness due to fast decay of Green's functions and sparseness due to the possible use of mutliresolution transformations. In conventional sparse solvers, the linear system is stored in its entirety using an appropriate sparse storage scheme, and a direct or iterative sparse-based technique is utilized for the inversion of the linear system. In this paper, a block decomposition of the moment matrix is proposed to take full advantage of all types and levels of sparseness plus any structure symmetries. Even though the proposed decomposition is quite intuitive, its implementation would not be feasible without resorting to object-oriented programming techniques. The computational efficiency of the proposed approach is demonstrated by numerical data.

II. MOMENT METHOD FORMULATION OF PLANAR STRUCTURES

In this section we summarize the moment method formulation of a generalized multilayered planar structure as depicted in Figure 1. This structure consists of several groups of material layers that are separated by ground planes. The electromagnetic coupling across the two sides of the ground planes is made possible through slot apertures. Using the equivalence principle, the ground planes are replaced by closed perfect conductors, and the slot traces are modeled by planar magnetic currents $\mathbf{M_s}$. The metallic traces are printed on the interface planes between material layers and are modeled by planar electric currents $\mathbf{J_s}$. The electric and magnetic currents constitute the computational domain of the integral equations of the problem.

To derive a system of integral equations for the planar structure, one can enforce the boundary conditions on tangential fields over metallic and slot traces in conjunction the electric, magnetic and mixed dyadic Green's functions [5]. In the method of moments, the currents are expanded in a proper set of basis functions such as generalized edge-based vector rooftop functions defined over rectangular or triangular cells or a combination of both. The cell equations are tested using Galerkin's method to obtain the following system of linear equations:

$$\begin{bmatrix} \mathbf{Z_{JJ}} & \mathbf{W_{JM}} \\ \mathbf{W_{MJ}} & \mathbf{Y_{MM}} \end{bmatrix} \begin{bmatrix} \mathbf{I_J} \\ \mathbf{V_M} \end{bmatrix} = \begin{bmatrix} \mathbf{V_J} \\ \mathbf{I_M} \end{bmatrix}, \qquad (1)$$

where $\mathbf{I_J}$ and $\mathbf{V_M}$ represent the amplitude vector of electric and magnetic currents, and $\mathbf{V_J}$ and $\mathbf{I_M}$ together represent the excitation vector. The matrix on the left side of equation (1) is the moment matrix made up of impedance submatrix $\mathbf{Z_{JJ}}$ (**J-J** interactions), admittance submatrix $\mathbf{Y_{MM}}$ (**M-M** interactions), and transfer submatrices (mixed interactions). A typical matrix element is given by the following equation:

$$Z_{mn}^{(\mu\nu)} = \iint_{S_m} ds \iint_{S_n} ds' \, \mathbf{f_m}^{(\mu)} . \overline{\overline{\mathbf{G}}}_{\mu\nu} . \mathbf{f_n}^{(\nu)}, \qquad (2)$$

where $\mathbf{f_m}^{(\mu)}$ and $\mathbf{f_n}^{(\nu)}$ represent the testing and expansion functions, respectively. Similar equations hold for the elements of the other submatrices.

III. TYPES OF MATRIX SPARSENESS

In equation (1), the moment matrix has been partitioned into submatrices identified by electric and magnetic testing and expansion functions. Using the equivalence principle, the traces above and below ground planes are naturally decoupled due to the placement of perfect conductor planes. Therefore, the corresponding segments of transfer submatrices $\mathbf{W_{JM}}$ and $\mathbf{W_{MJ}}$ contain only zeros. To take full advantage of these zero elements, the moment matrix should be rearranged according to the natural order of the traces on the substrate structure. This is shown schematically in Figure 2. The moment matrix is effectively reduced to a block diagonal matrix. The overlapping dark areas represent the interactions between magnetic currents on the same ground plane, which indeed provide the electromagnetic coupling from one side to the other. The dashed squares represent the interactions among electric currents on one side of a ground plane.

It is well known that the Green's functions of a multilayered substrate structure decay as a function of the distance between the observation and source points. The Green's functions have source singularities that need to be removed in order to have converging moment integrals. To facilitate the convergence of the integrals, a mixed potential formulation is often adopted. This method makes use of vector and scalar potentials with reduced singularities. Considering the interactions among the basis functions on the same trace plane or on different trace planes, the following inequality holds:

$$\left| Z_{mn}^{(\mu\nu)} \right| \approx \Psi_{\mathbf{m}}^{(\mu)} . \left| \overline{\overline{\mathbf{G}}}_{\mu\nu}(R_{mn}) \right| . \Psi_{\mathbf{n}}^{(\nu)}, \quad R_{mn} \to \infty, \qquad (3)$$

where R_{mn} is the distance between the centers of the testing and expansion functions, and $\Psi_m^{(\mu)}$ is the moment of the basis function. Similar inequalities hold for the elements of the other submatrices. Based on the distance between basis functions and the rate of decay of Green's functions, it is thus possible to find bounds for moment matrix elements as they move away from the diagonal of the matrix. If the basis functions are arranged in structured groups, knowing the maximum cell sizes, one can estimate the bounds for the interaction between any two groups.

The third type of sparseness is encountered when using multiresolution techniques in conjunction with the method of moments. Due to the vanishing moment property of wavelet basis functions and the resulting cancellation effect, a wavelet-dominated moment matrix tends to be highly sparse. For rectangular structures, wavelet expansion or transformation techniques usually provide a high degree of matrix sparseness [2]. However, the application of wavelet techniques to nonrectangular geoemtries has not yet been reported to this date.

IV. BLOCK DECOMPOSITION OF MOMENT MATRIX

The metallic and slot traces printed on the trace planes or ground planes of a planar structure are indeed made up of geometrical objects that are connected to one another. These objects are discretized using a proper meshing scheme, upon which the elementary electric and magnetic currents represented by basis functions are built. In an object-oriented implementation of the method of moments, the geometrical objects are encapsulated as software objects with their own graphical, physical and meshing properties. The continuity of currents among the geometrical objects is provided through "connection objects". Each object represents a structured group of electric or magnetic rooftop basis functions.

In an object-oriented programming context, the moment matrix can be viewed as a collection of block matrices containing the interaction among various groups of basis functions. The blocks are created as necessary during the computation of moment interactions. For example, a block representing the interaction between two groups of basis functions located on the two different sides of a ground plane is naturally zero and does not need to be stored in the memory. Moreover, using the symmetry or reciprocity properties, identical or similar conjugate blocks across the diagonal are stored only once. Any thresholding process due to decay of Green's functions or multiresolution schemes is performed block by block, and the resulting sparse submatrix is stored in a one-dimensional row-indexed sparse array. Not only does this approach lead to a significant amount of saving in memory usage, but it also drastically speeds up the operation of matrix-vector multiplication:

$$\mathbf{A}.\mathbf{x} = \left[\sum_{i=1}^{N} \mathbf{A}_{ij} . \mathbf{x}_j \right]_i , \tag{4}$$

where $\mathbf{A}_{ij}$ are the blocks comprising the moment matrix. The summation of equation (4) runs only through the "nonzero" blocks of the moment matrix. All identical or conjugate blocks have a common pointer to the pertinent stored memory addresses, which are called upon as necessary. The block matrix-vector multiplications on the right hand side of equation (4) are performed using a row-indexed sparse scheme. Utilizing an efficient sparse iterative solver such as a preconditioned bi-conjugate gradient method (BiCG), the block matrix-vector multiplication of equation (4) can be implemented very effectively.

IV. NUMERICAL RESULTS

The object-oriented sparse MoM solver described in this paper has been tested for a variety of large-scale planar structures. In this synopsis, we present the results for the full-wave simulation of the corporate feed network of an eight-element patch array as shown in Figure 3. The patches are 40.2mm squares uniformly spaced at a distance of 53.6mm from each other. The circuit is printed on a 1.59mm thick grounded substrate with ε_r=2.57. The microstrip lines are all 0.446mm wide with mitered bends. The objective is to find the input reflection coefficient of the feed network, which is -0.18-j0.518. Table 1 indicates the computation time and memory usage of the simulator with three different solvers: (1) a direct solver based on LU decomposition, (2) an iterative solver based on conventional BiCG method, and (3) the object-oriented sparse solver proposed in this paper. All three cases were run on a 300MHz Pentium II personal computer running under Windows NT. The size of the linear system in this case is 2194. The advantage of the proposed method over a conventional BiCG solver is evident from a four-fold improvement in computation time and a ten-fold saving in memory usage. A very low threshold level of 10^{-6} was used for the sparse scheme. Even though the direct solver performs relatively well in view of computation time, its speed and memory usage will adversely deteriorate as the size of the linear system increases slightly. More numerical data for various planar circuits will be presented at the symposium.

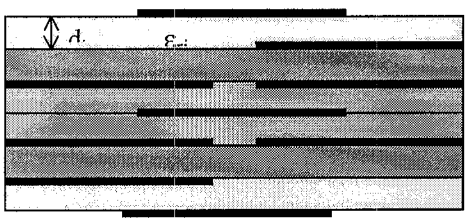

Figure 1. A general multilayered planar structure.

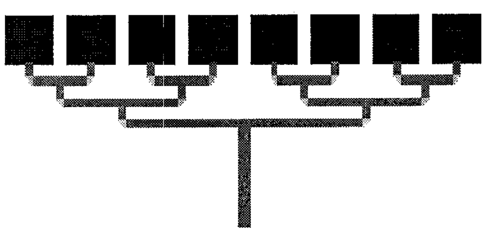

Figure 3. Corporate feed network of an 8-element patch array.

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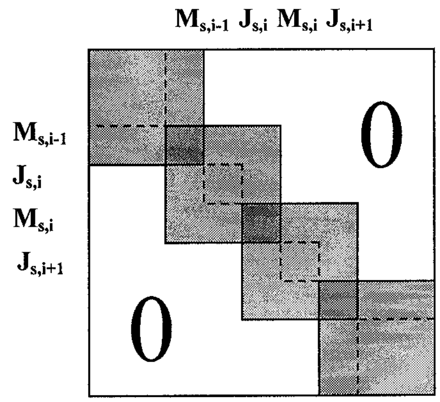

Figure 2. Rearrangement of moment matrix in a block diagonal form.

Table 1

	Proposed Approach	Conventional BiCG	Direct LU Decomposition
Matrix Fill Time [s]	144	144	144
System Inversion Time [s]	453	1923	733
Total Computation Time [s]	602	2072	882
Memory Usage [MB]	7.5	77	77